Projectiles at an angle off a cliff

GAZELLES OUT OF CANNONS OFF CLIFFS

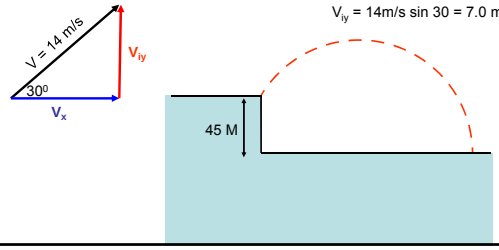
Sample Problem

An arrow is shot at an angle of 30 degrees above horizontal from a cliff that is 45 meters tall. The arrow leaves at a speed of 14 m/s. How much time to reach maximum height above cliff? What is maximum height above the cliff? What is the total time the arrow is in the air? How far from the base of the cliff does the arrow land?

FIRST DRAW A PICTURE AND RESOLVE THE VELOCITY INTO X AND Y COMPONENTS:

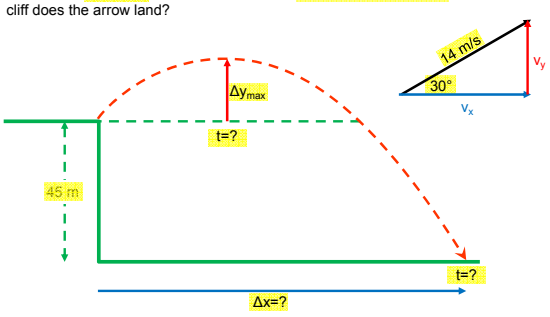
V_x = 14m/s cos 30 = 12.12 m/s

V_{iy} = 14m/s sin 30 = 7.0 m/s

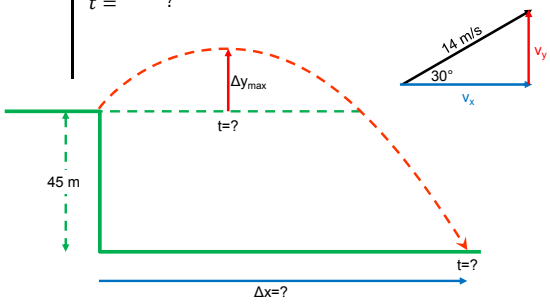


Sample Problem 1

An arrow is shot at an angle of 30 degrees above horizontal from a cliff that is 45 meters tall. The arrow leaves at a speed of 14 m/s. How much time to reach maximum height above cliff? What is maximum height above the cliff? What is the total time the arrow is in the air? How far from the base of the cliff does the arrow land?

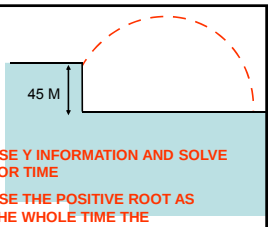


x	y
$\Delta x =$	$\Delta y = -45\ m$
$v_x =$	$v_i = 7$
$t =$	$v_f =$?
	$a = -9.8\ m/s^2$
	$t =$?



SPLIT INFORMATION INTO X AND Y:

X	Y
V_x = 12.12 m/s	V_i = 7.0 m/s
ΔX = ?	a = -9.8 m/s²
t = ____	ΔY = -45 m
	t = 3.83 sec



USE Y INFORMATION AND SOLVE FOR TIME

USE THE POSITIVE ROOT AS THE WHOLE TIME THE ARROW SPENDS IN THE AIR WITH X INFO:

Solve:

$$\Delta Y = V_i t + \frac{1}{2}at^2$$

$$-45m = (7.0m/s)t + \frac{1}{2}(-9.8m/s^2)t^2$$

$4.9t^2 - 7t - 45 = 0$ **Quadform**

$t = 3.83\text{sec}$ $\quad t = -2.4\text{sec}$

How Far ?

$$V_x = \frac{\Delta x}{t}$$

$$12.12m/s = \frac{\Delta x}{3.83\text{sec}}$$

$$\Delta x = 46.42m$$

Now look at the questions. . .

- how much time to maximum height above cliff? Hmmmmm. . . .
  - ➢ the positive root was TOTAL time. . . 3.83 sec
- What is y velocity when it returns to cliff level?
  - ➢ -7 m / s (this is conceptual. . .no math needed)

- Maximum height above cliff?
  - Time to maximum height is 0.72 sec
  - V_{iy} = 7.0 m/s
  - a = -9.8m/s^2

$$\Delta y = V_i t + \frac{1}{2}at^2$$

$$\Delta y = (7.0m/s)(0.72s) + \frac{1}{2}(-9.8m/s^2)(0.72s)^2$$

$$\boldsymbol{\Delta y = 2.50m}$$

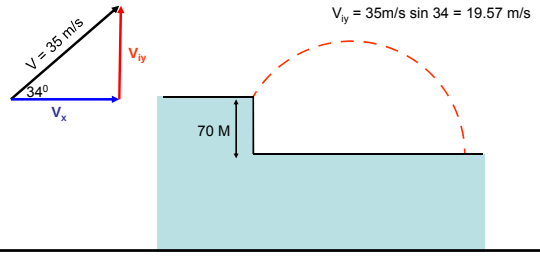
Sample Problem

A gazelle is launched from a 70 meter high cliff at 35 m/s at and angle of 34 degrees above horizontal. How long, was the gazelle in the air? How far from the base of the cliff did the gazelle land? How high relative to the ground did the gazelle go?

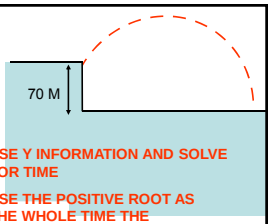
FIRST DRAW A PICTURE AND RESOLVE THE VELOCITY INTO X AND Y COMPONENTS:

V_x = 35m/s cos 34 = 29.02 m/s

V_{iy} = 35m/s sin 34 = 19.57 m/s



SPLIT INFORMATION INTO X AND Y:



X	Y
V_x = 29.02 m/s	V_i = 19.57 m/s
ΔX = ?	a = -9.8 m/s^2
t = ____	ΔY = -70 m
	t = 6.27 sec

USE Y INFORMATION AND SOLVE FOR TIME

USE THE POSITIVE ROOT AS THE WHOLE TIME THE ARROW SPENDS IN THE AIR WITH X INFO:

Solve:

$$\Delta Y = V_i t + \frac{1}{2}at^2$$

$$-70m = (19.57m/s)t + \frac{1}{2}(-9.8m/s^2)t^2$$

$4.9t^2 - 19.57t - 70 = 0$ **Quadform**

$t = 6.27\text{sec}$ $\quad t = -2.28\text{sec}$

How Far ?

$$V_x = \frac{\Delta x}{t}$$

$$29.02m/s = \frac{\Delta x}{6.27\text{sec}}$$

$$\Delta x = 181.96m$$

How high relative to the ground?

X	Y
V_x = 29.02 m/s	V_i = 19.57 m/s
ΔX = ?	a = -9.8 m/s^2
t = ____	ΔY = ?
	t = ____sec

Quadform: positive root = 6.27 sec; negative root = - 2.28 sec

Time above cliff 6.27 – 2.28 = 3.99 sec

Time to max height is 3.99/2 = 1.995 sec

$$\Delta Y = V_i t + \frac{1}{2}at^2$$

$$\Delta Y = (19.57m/s)1.995 + \frac{1}{2}(-9.8m/s^2)(1.995)^2$$

$$\Delta Y = 39.04 - 19.50$$

$$\Delta Y = 19.54m$$

70 m + 19.54 m = 89.54 m (relative to the ground)